Efficient Algorithms for Semirandom Planted CSPs at the Refutation Threshold

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Joint work with

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CMU

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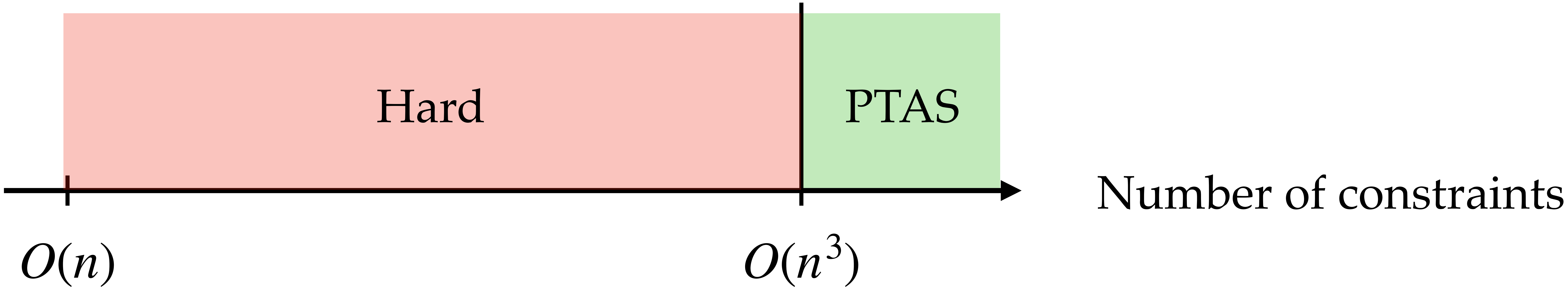

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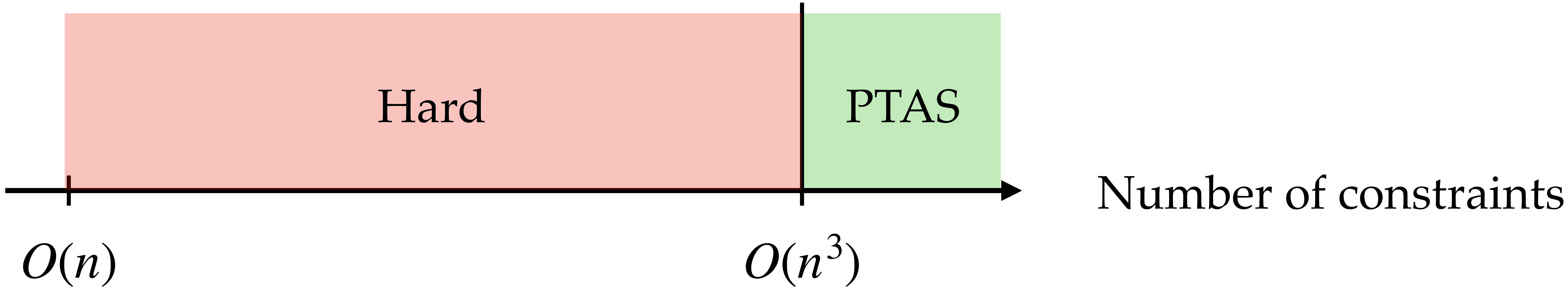

Worst-case setting.

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Fact: All assignments satisfy $\frac{7}{8} \pm o(1)$ fraction of the clauses.

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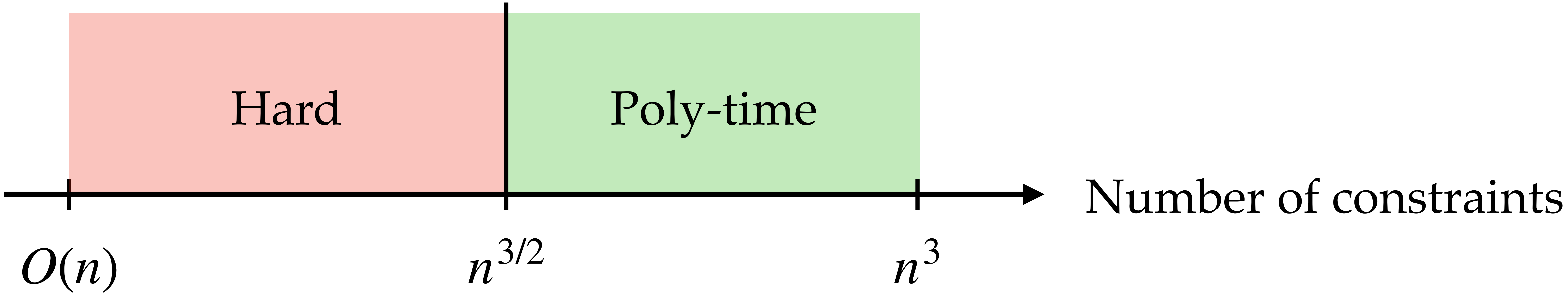

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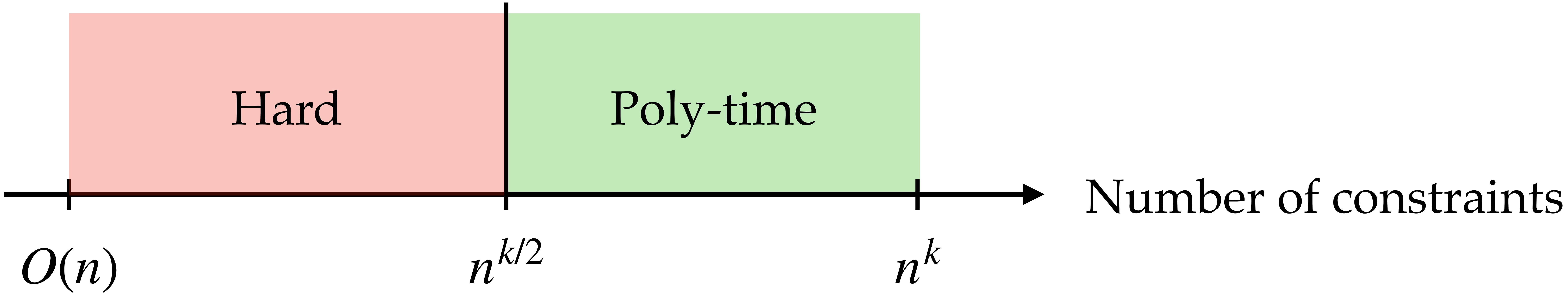

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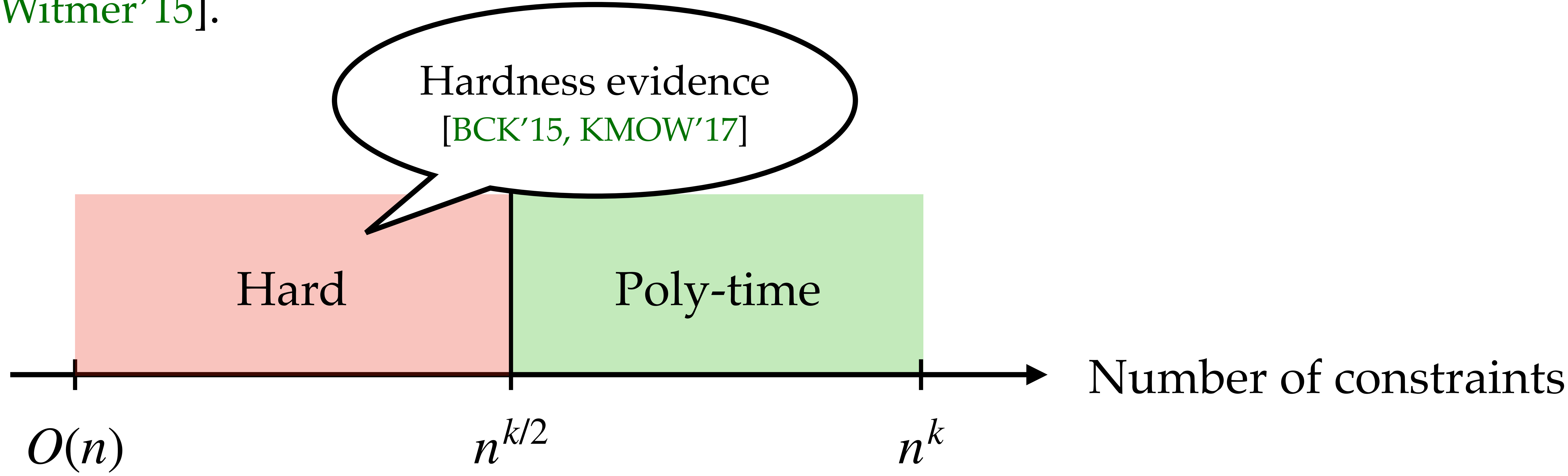

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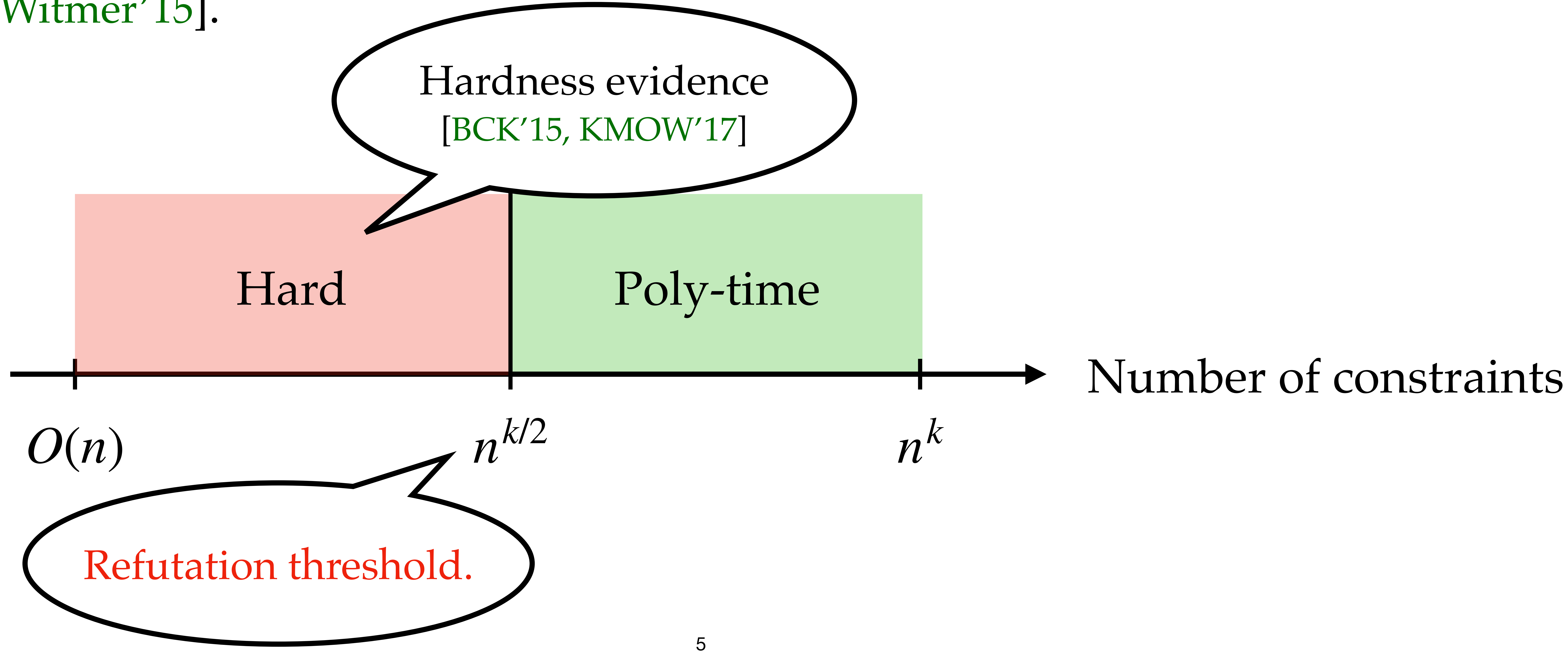

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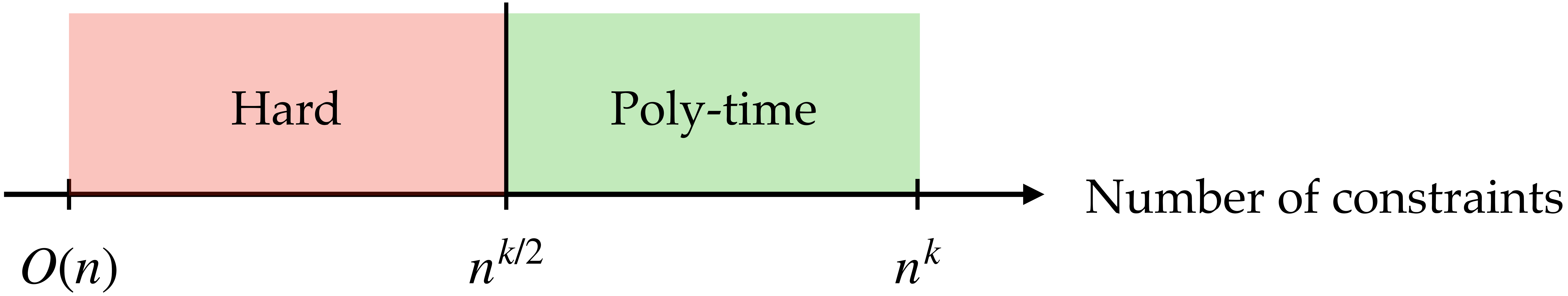

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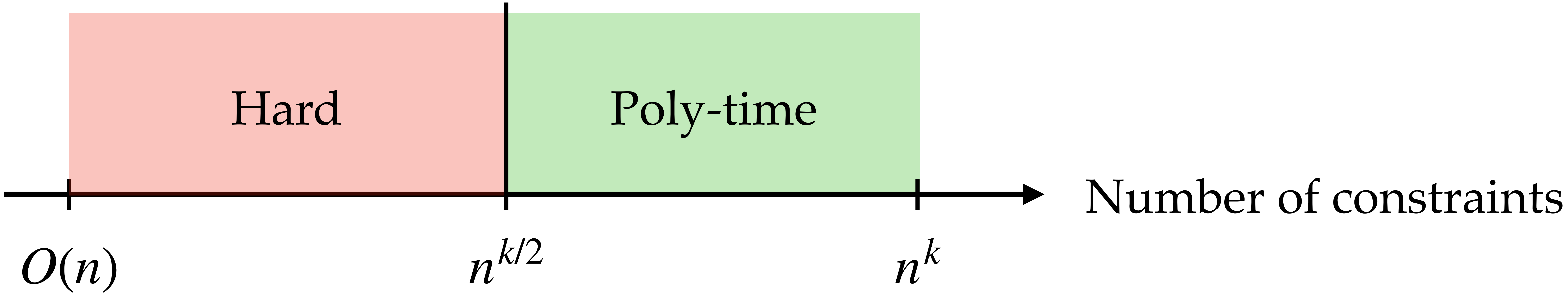

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- Algorithms that succeed are more "robust".
- Understand which "randomness" is unnecessary.

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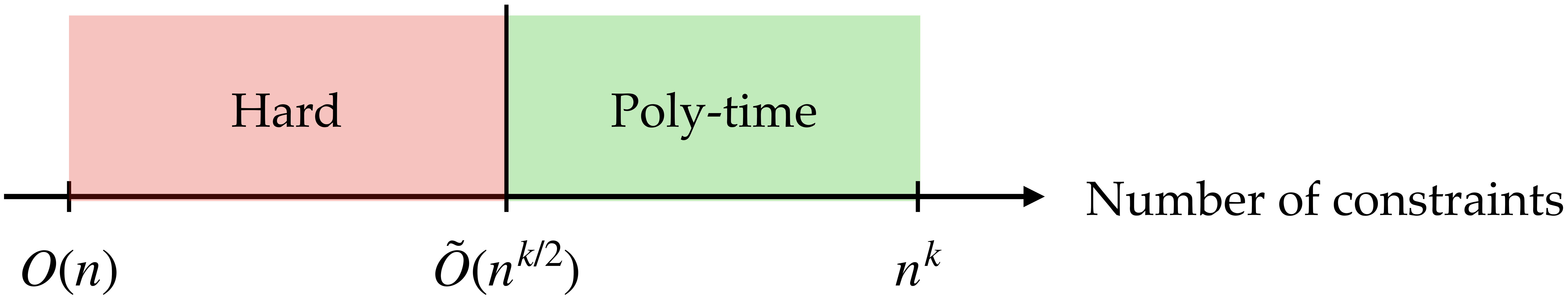

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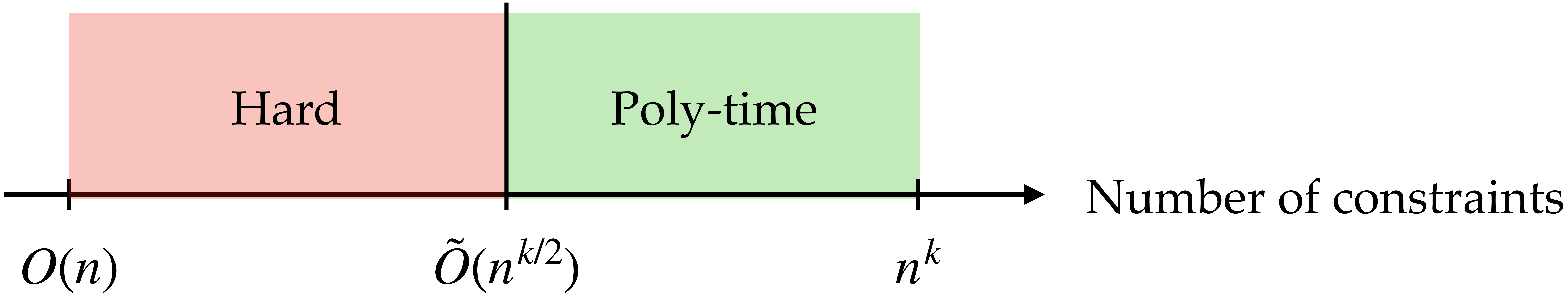
Search (planted): ?



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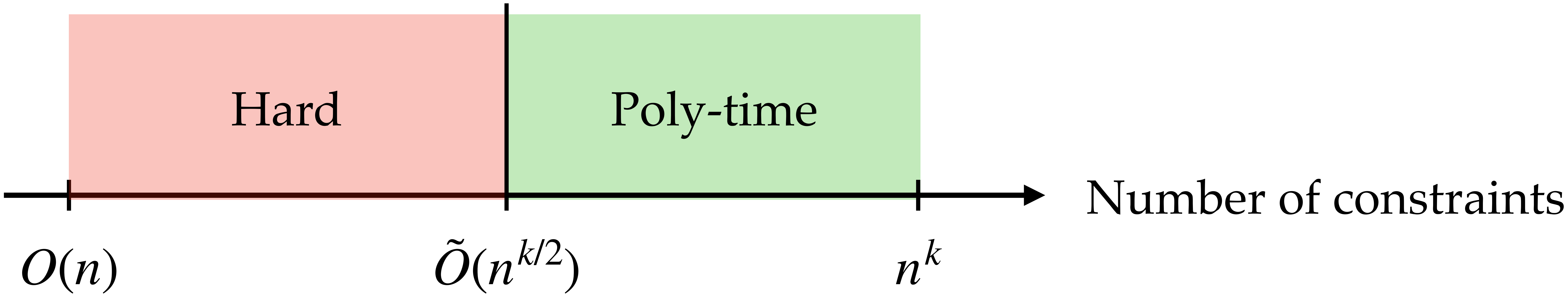

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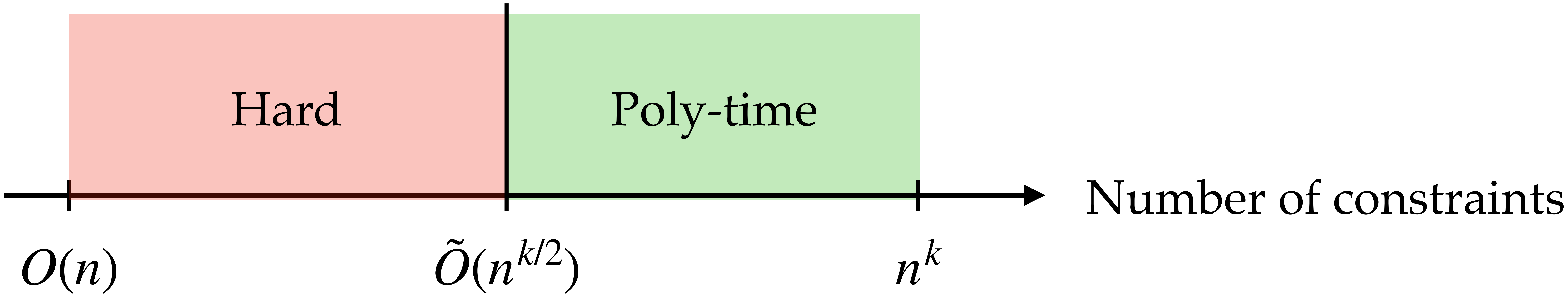

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Fourier analysis.

Noisy planted k-XOR

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Our result: we identify **almost all** corrupted constraints.

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This stronger guarantee is necessary for
the reduction from semirandom k-CSPs to noisy k-XOR.

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Decomposition that breaks up every instance into pieces satisfying the above condition.

Proof Idea:

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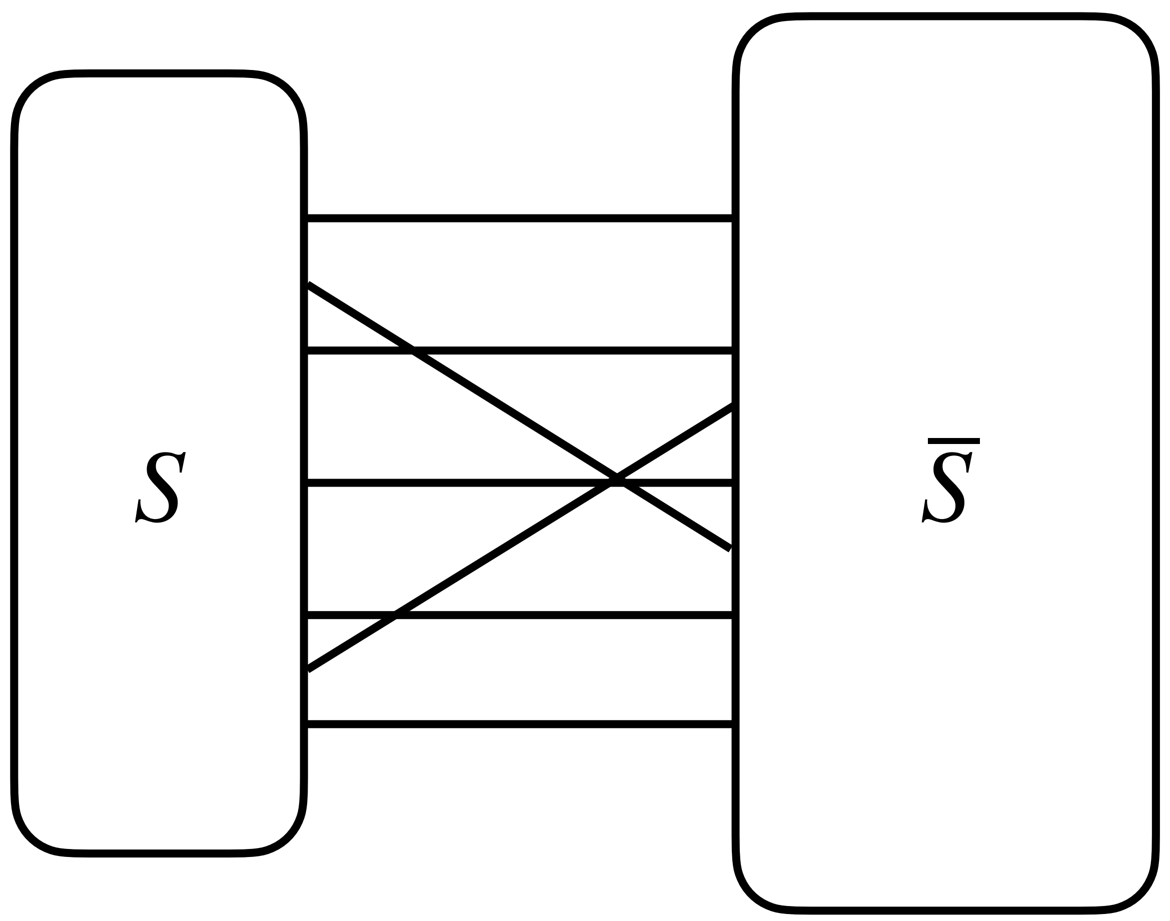

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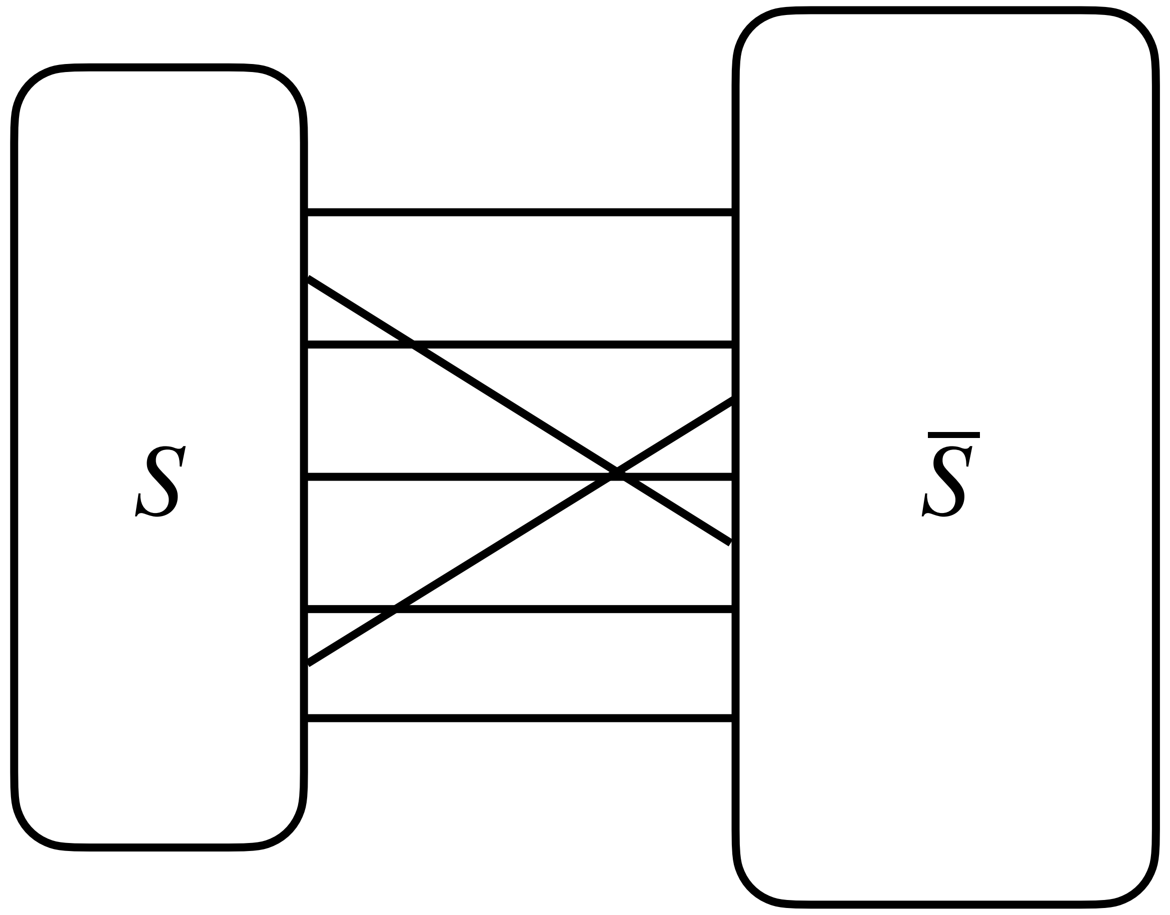

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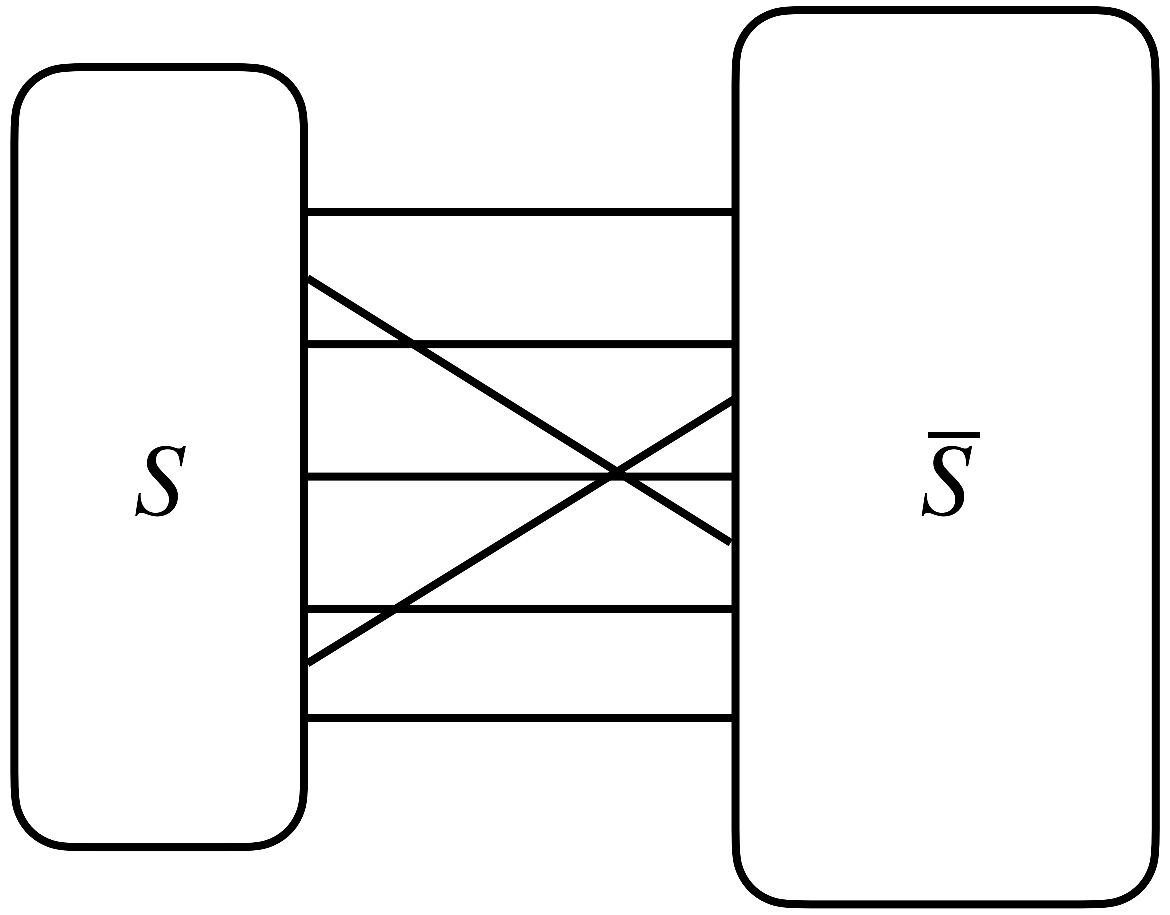

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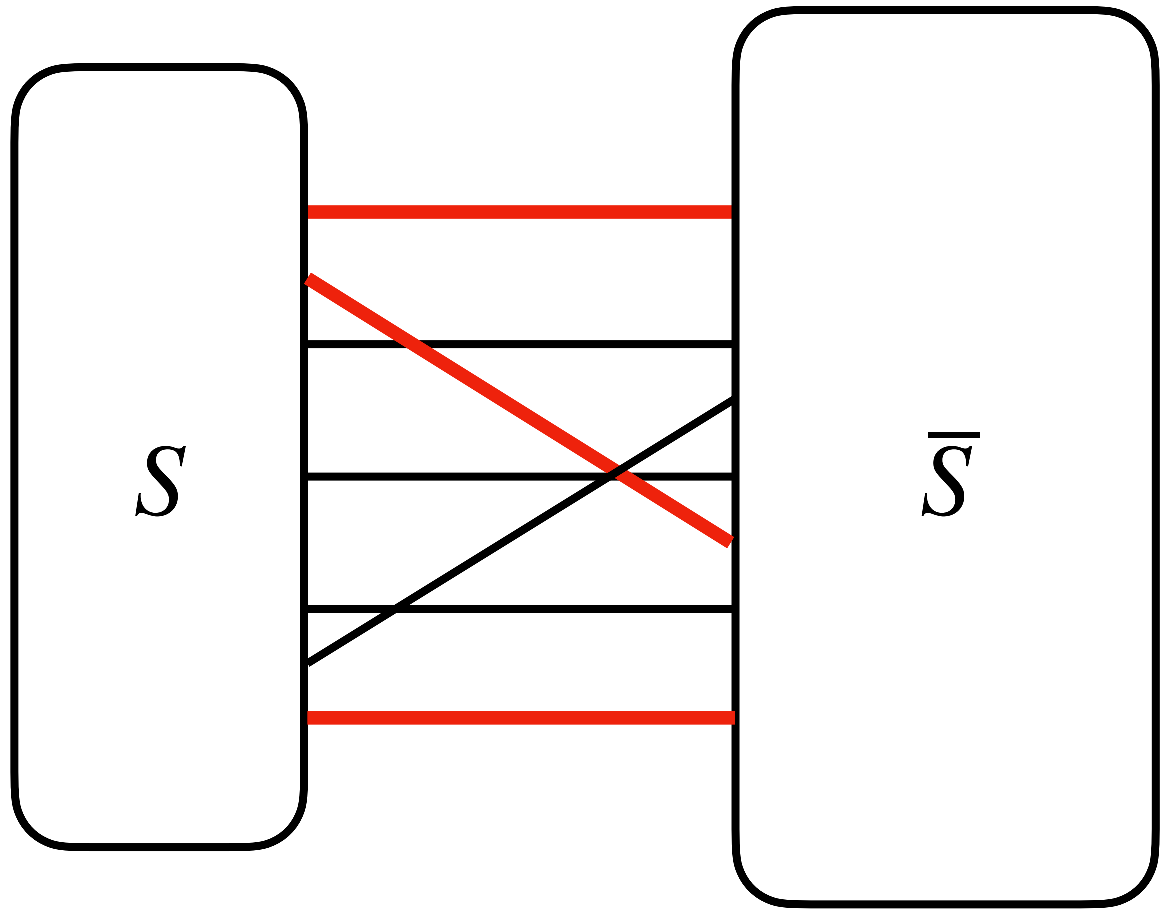

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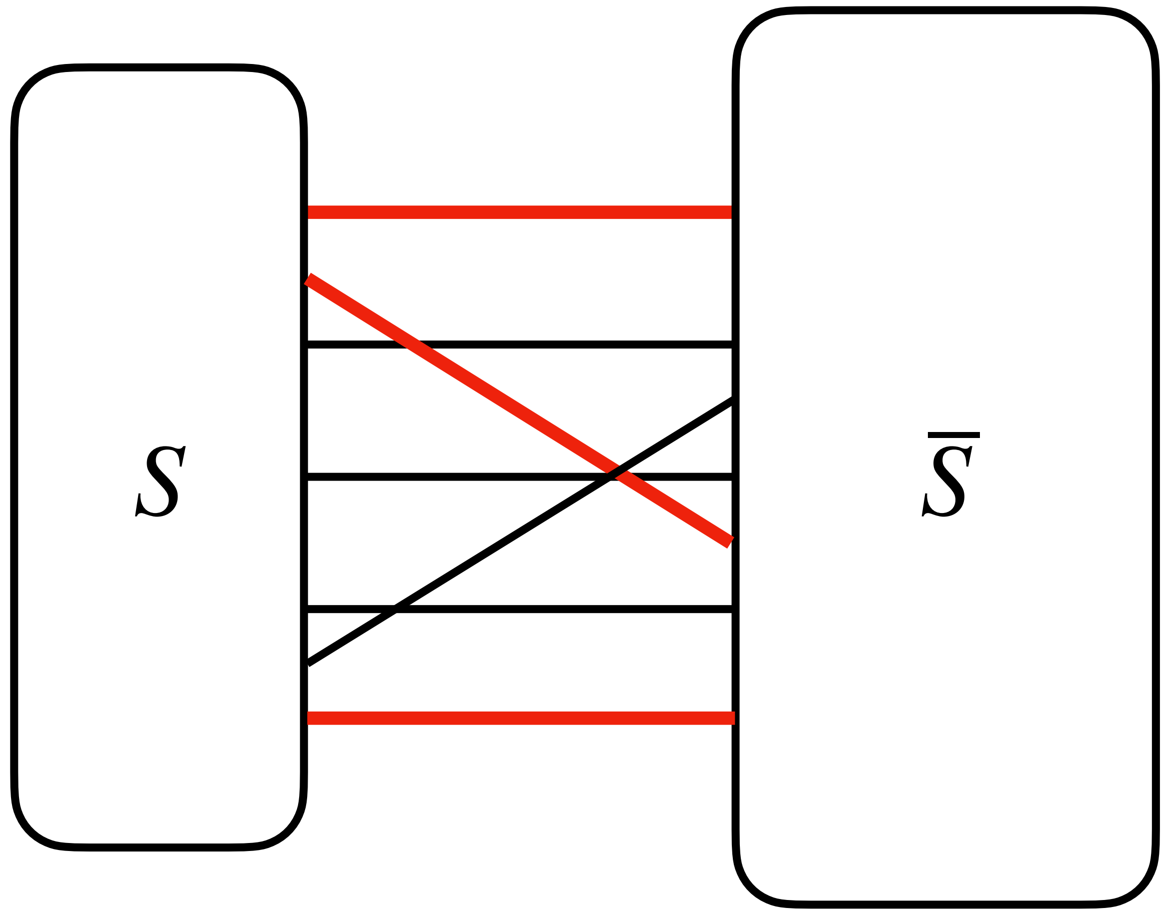

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Need additional assumptions.

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Spectral sparsification

Lemma. $L_H \prec \frac{1}{2} L_G \implies X = x^* x^{*\top}$ is the unique optimal SDP solution.

Lemma. Suppose G has spectral gap λ and min-degree d such that $\lambda d \geq \omega(\log n)$, then H is a spectral sparsifier of G (w.h.p.), meaning

$$L_H \preceq (1 + o(1)) \cdot \eta \cdot L_G.$$

Large spectral gap + min degree $\implies x^* x^{*\top}$ is the unique SDP optimum.

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  - recover x^* and identify corrupted edges in each sub-instance.

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$\longrightarrow$ a generalized version of spectral sparsification.

Conclusion

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Thank you!

https://arxiv.org/abs/2309.16897